A numerical approach to the keel design of a sailing yacht

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**Article Info**

**Abstract**

This paper describes an approach to the keel design of a sailing yacht. The related software, which is fully automatic, leads to the optimal shape by modifying the surface used to define the keel planform. B-spline curves and surfaces have been used due to their ability in following complex shapes. The algorithm integrates ad hoc implemented original software with computational fluid dynamics (CFD) commercial ones. The optimisation procedure uses genetic algorithms (GAs) and a gradient-based optimiser for the refinement of the solution. A careful CAD and CFD modelling leads to a stable and efficient generalised method, which has been applied to the design of the centreboard of the 5o5 international class racing dinghy.

**Keywords:**

Sailing yacht, CFD, GAs, Design optimisation

**1. Introduction**

Keel design is not a trivial task; the evidence of this is that there is still not a close analytical solution of the problem. Increased computational capabilities allow executing a large number of numerical simulations postponing the more expensive experimental test (which still remains fundamental to validate numerical results) only on a limited number of solutions. For this reason, nowadays it is common practice to use computational fluid dynamics (CFD) software based on RANS (Reynolds-averaged Navier–Stokes equations) (Mancuso and Mancuso, 2004; D’Anca et al., 2005), instead of the less time-consuming potential flow-based methods (Valorani et al., 2003), or simpler ones (Percival et al., 2001). As a matter of fact (Parolini and Quarteroni, 2005), it must be observed that in complex optimisation problems, a potential flow solver can be employed in the early stages of the design; once a rough solution is found, this one can be refined with the most accurate method based on RANS. In each case, it is well known that such numerical results must be carefully interpreted because of unavoidable approximations of the numerical simulations. Moreover, concerning sailing yachts, in many cases the real sailing conditions are difficult to predict and/or simulate for several reasons. For example, the heel angle of a yacht sailing close hauled is not always taken into account as in Yoo and Kim (2006) concerning sails and in Poloni et al. (2000) concerning keels; neglecting this angle can lead to wrong or inaccurate results. In other cases, the Reynolds number falls in the transition between laminar and turbulent flow, so that, many turbulent models fail in predicting the fluctuating components.

However, the comparison of different numerical solutions (often indicated as benchmark test) can give reasonable results due to bias behaviour of the simulation error (Oberkampf and Trucano, 2002), if the standpoint is the comparison between results instead of the evaluation of absolute values. For this reason, in the last years, optimisation methodologies have been intensively applied to solve complex fluid dynamics problems. Particularly, at the beginning most of the applications, not only concerning CFD, were addressed to the study of International America’s Cup Class Yacht (Fiddes and Gaydon, 1996; Burns Fallow, 1996; Charvet et al., 1996; Miyata and Youn-Woo, 1999; Harries et al., 2001) for which the syndicates budget was enough to sustain the research cost. Once, these resources have become more accessible to the scientific community, it has been possible to extend studies even in other fields of interest (Hochkirk et al., 2002; Carswell and Lavery, 2006).

In this paper, a strategy to find an optimal keel planform has been developed. It consists of three linked steps. At first, the CAD model of the keel has been defined by means of non-uniform rational B-spline (NURBS) curves and surfaces because of their ability in following complex shapes. In fact, the local support property of the NURBS (Piegl and Tiller, 1997) allows to model a complex surface like the hull (Percival et al., 2001) or surfboard fins (Carswell and Lavery, 2006) with fewer parameters than Bézier surfaces, and thus avoiding the addition of perturbation surfaces as in Valorani et al. (2003), or the introduction of additional structures which control the Bézier points as in Lombardi et al. (2007). Then, the CFD model both in terms of geometry and simulation has been set up finding a suitable structure for domain and meshing parameters; in fact, as explained later, the use of adequate blocking topology as in Yoo and Kim (2006), gives rise to a hexahedral grid whose simulation leads to more accurate results than those obtained with a
tetrahedral one. Finally, these two steps have been integrated into ad hoc developed optimisation software based upon genetic algorithms (GAs) and a gradient-based optimiser for the solution refinement. Fig. 1 shows the algorithm flow-chart. In the next sections, the three steps will be detailed referring to the specialised literature for further details (Vanderplaast, 1984; Goldberg, 1989; Kennicott, 1996; Piegl and Tiller, 1997).

2. Centreboard design: classical approach

The importance of a well-engineered centreboard for a modern sailing yacht can be summarized by this consideration: without the centreboard a modern sailing yacht (except multi-hulls) can efficiently sail with an angle, between boat heading and wind direction, of about 130–150°. Considering that most regattas have at least an upwind leg and that it is often fundamental, the time spent by the engineers to design a suitable centreboard is justified. The centreboard contributions were evident after the engineers to design a suitable centreboard is justified. The centreboard contributions were evident after the aerodynamics study at the beginning of the 20th century when the equilibrium between aerodynamic and hydrodynamic forces became well known (Garrett, 1987). Classical aerodynamics theory predicts, for a high efficiency wing with finite aspect ratio, a planform that provides an elliptical lift distribution. The theory predicts, for a high efficiency wing with finite aspect ratio, it is possible to calculate the aerodynamic resultant force one based on direct experience. Starting from those values, it is possible to calculate the aerodynamic resultant force (sail contribution only), for example by means of the Momentum variation approach (Garrett, 1987):

\[
F = \rho A v^2 \frac{1}{2} \times (1 - \cos \theta),
\]

(1)

\[
L_S = F \times \sin(\beta + \theta + \lambda),
\]

(2)

\[
\beta = \tan^{-1} \left( \frac{\sin \theta}{1 - \cos \theta} \right),
\]

(3)

where \( F \) is the resultant aerodynamic force, \( \rho \) is the air density, \( A \) is the total sail area (main+jib), \( \theta \) is the deflection angle of the air flow, \( L_S \) is the heeling component provided by the sails, \( v_a \) is the relative wind velocity, \( \beta \) is the angle between force \( F \) and apparent wind direction, and \( \lambda \) is the boom angle.

Fig. 2. The equilibrium between aerodynamic and hydrodynamic forces.

If experimental data (e.g., wind tunnel) are available, the heeling force, \( L_s \), can be calculated by means of the following equation:

\[
L_S = \frac{1}{2}\rho v^2 A \times C_{H}(\alpha),
\]

(4)

where \( C_{H}(\alpha) \) is defined as

\[
C_{H}(\alpha) = C_L(\alpha) \times \cos(\alpha) + C_D(\alpha) \times \sin(\alpha)
\]

(5)

being \( C_L(\alpha) \) and \( C_D(\alpha) \) the experimental lift and drag coefficients for a given apparent wind angle \( \alpha \).

Once the sail heeling force has been obtained, it is possible to calculate the necessary centreboard area. In fact, choosing the leeway angle and the centreboard cross-section, the lift coefficient for those conditions, can be found in literature.
planform surface is calculated, for instance, by means of the classical lift equation (Eq. (6)):

\[ L_C = \frac{1}{2} \rho_w V^2 S \times C_L, \]  

(6)

where \( L_C \) is the lift component provided by the centreboard (equal to the heeling component provided by the sails assuming no lift from hull and rudder) which has been assumed equal to the heeling component due to small leeway angles, \( \rho_w \) is the water density, \( V \) is the boat velocity, \( S \) is the centreboard planform area, and \( C_L \) is the lift coefficient.

Afterward the planform is designed to stay in the trunk and to have an elliptical lift distribution that, from classical aerodynamic literature, produces the lowest drag. However, real sailing conditions are generally far away from the hypothesis of the classical aerodynamics theory, thus this approach often leads to unsatisfactory results.

For these reasons, in the last years, the availability of commercial software that conveniently works even on personal computer permits a numerical approach to the design.

3. Centreboard design: numerical approach

As previously outlined, the keels of sailing yachts are designed according to aerodynamics theory. Even if this approach can be discussed, it represents a valid starting point. In fact, the analogy between air and water, if well simulated, permits to establish the correct similitude and thus to apply typical aerodynamic concepts to hydrodynamics. For this reason, cross-sections and planform are obtained from the lift theory (Abbott and Von Doenhoff, 1959). For many of these sections, a mathematical formulation exists which, for practical needs, is converted in a suitable format for CAD applications.

3.1. CAD modelling

The choice of the section represents a crucial point in the keel design. The most popular both for keel and rudder are the NACA four digit. Thus, the point sequence obtained by the mathematical formulation has been converted into a NURBS curve by means of approximation techniques (Cappello and Mancuso, 2004). Once the section has been defined, the surface can be obtained by means of a tensor product surface (Piegl and Tiller, 1997), having \( M \) sections each with \( N \) control points, \( M \) and \( N \) being adapted for each problem. Moreover, many software (e.g., CAD, CFD, FEM) accept as an input the IGES format (Kennicott, 1996), and the easy and manageable structure of this ASCII file permits us to avoid many data exchange failures which may occur if it contains (as typically happens) redundant or unnecessary information. For this reason, the implemented software automatically saves the IGES file with the necessary information only. Fig. 3 shows section and planform of a centreboard having \( M = 5 \) and \( N = 50 \).

3.2. CFD modelling

Even if the model geometry is a relatively simple composition of four NURBS surfaces only (upper and lower surfaces, tip surface and trailing edge surface), the goal is to define a structure for the domain able to guarantee, with the lowest number of elements, a good centreboard shape approximation and, at the same time, reliability of the results. For this reason, preliminary 2D analyses on the cross-section of the centreboard have been executed (Barraco et al., 2007) aiming at set up correct simulation parameters (e.g., turbulence model, discretisation scheme, under relaxation factors). Numerical results in terms of lift and drag coefficients have been compared with Abbott and Von Doenhoff.
finding a difference, in the range 2–10° of the angle of attack, less then 1% for \( C_L \), and 4% for \( C_D \).

Concerning the 3D discretisation (Fig. 4, left), a blocking strategy (by means of edge and face projection without O-Grid) has been adopted; a single fluid block has been divided into 13 hexahedral sub-blocks (whose size is a parametric function of the centreboard main dimensions). The enlarged view of Fig. 4 shows how the blocks of the centreboard wall are linked to the Cartesian coordinates of the surface control points used to split the grid in the top-down direction. This approach has several advantages. At first, different geometries (in terms of span and chord) can be modelled with only one topology; then, the mesh follows the leading and trailing edges geometries (in terms of span and chord) can be modelled with only one topology; then, the mesh follows the leading and trailing edges with more accuracy (even if they have a complex shape) because the number of wall elements can be adjust by the user; finally, the meshing parameters in the fore and aft direction can be easily adjusted according to the flow characteristics. Fig. 5 (left) shows how the blocks of the centreboard wall are linked to the z-coordinates of the surface control points to which apply the dimensional scaling, \( S_j \), and translation, \( T_j \). (DV1–DV4), five section scaling \( S_j \) (DV5–DV9) and one span scaling \( s \) (DV10). On the right side of Fig. 6, the initial set of design variables corresponding to a trapezoidal planform (DV = [0 0 0 0 1 1 1 1 1]) and reasonable lower and upper bounds are shown.

4.2. Objective function

The formulation of the objective function is, in most cases, a difficult and, at the same time, fundamental task. It must combine both theoretical and empirical concepts. From the theoretical point of view, sailing close hauled the centreboard has to act as a pressure relief, would be too complex to be included in the optimisation routine. Hence, the optimum planform may vary from elliptical if this effect is included.

\( \text{Objective function value. Generally, for engineering applications, the design variables are bounded and thus may change inside a domain which constitutes the so-called Design Space.} \)

\[ \begin{align*}
\text{CP}_{ij}^T &= (\text{CP}_{ij}^S \times S_j) + T_{j-1} \\
\text{CP}_{ij}^S &= \text{CP}_{ij}^T + S_j \\
\text{CP}_{ij}^F &= \text{CP}_{ij}^T \times s \\
\text{CP}_{ij}^F &= \text{CP}_{ij}^T \times S_j \\
\text{CP}_{ij}^F &= \text{CP}_{ij}^T \times T_j \\
\text{CP}_{ij}^F &= \text{CP}_{ij}^T \times T_0 = 0
\end{align*} \]
4.3. Optimisation algorithms

The optimisation process uses a GA (Cappello and Mancuso, 2003) and a constrained gradient-based algorithm for the refinement of the solution.

4.3.1. GAs structure

In a GA, each of the design variable is encoded as a string composed over some alphabet(s), for example, binary, integer, real valued. The strings corresponding to a variable set are gathered in a structure like the chromosomal one (individual), whose genes are the bits in case of binary codification of the variables, or the variables themselves in case of decimal or integer format. The GA produces an evolutionary process on an initial population (Goldberg, 1989). It is based on the calculation of a fitness for each individual, depending on the value of the objective function. The procedure is repeated until satisfying results have been achieved or the improvement halts or the algorithm has executed the prescribed iterations. Differently from the conventional algorithms, the GAs act simultaneously on points belonging to many regions of the design domain, so that the risk to find local optima is reduced. Moreover, only information concerning the objective function are used, differently from the so-called first-order methods (Vanderplaast, 1984) that also require the calculation of the first derivative slowing the process and reducing the possibility to explore each region of the domain from the given starting point.

4.3.1.1. Selection phase. As in nature, individuals with better fitness have a better probability of surviving and reproducing. The individual selection is controlled statistically by two parameters calculated for each individual probability of crossover, \( p_c \), and probability of mutation, \( p_m \). Two coefficients, \( r_c \) and \( r_m \), are randomly drawn for each individual; the individual is selected for crossover if \( r_c < p_c \) and for mutation if \( r_m < p_m \). Increasing \( p_c \) and \( p_m \), the probability that the individual is selected increases.

4.3.1.2. Crossover and mutation phases. Two individuals selected for crossover transmit their genes to two new individuals obtained by their gene combination, which will replace the first two. The combination can happen through one or more points by interchanging the parts located at the same side of the points.

With mutation the value of a gene is randomly changed, varying a bit in binary codification case, or a variable in other cases. Therefore, mutation allows the random shifting in the domain by exploring both different and new regions. Moreover, it inserts a degree of genetic diversity in the solutions, useful when the algorithm tends to produce individuals thickened in a convergence region. Thus, the population, which is randomly generated inside the design space and keeps constant size, evolves owing to the modifications performed by the operators of crossover and mutation.

4.3.2. Constrained optimisation

In order to refine the local optimal solution obtained with the GAs, a constrained gradient-based optimisation (Mancuso, 2006), can be applied. The problem has been solved with the aid of the optimisation toolbox of MatLab (Matlab R13, 2002). In particular, efficient algorithms are made available to the optimiser by the BFGS (Broyden–Fletcher–Goldfarb–Shanno) quasi-Newton method, with a mixed quadratic and cubic line search procedure. This quasi-Newton method uses the BFGS formula to update the approximation of the Hessian matrix, since no information about the gradient of the objective function can be supplied (a detailed description can be found in Matlab R13, 2002). Particularly, lower and upper bounds of the variables (like those of Fig. 6) can be reduced close to the local solution. Care must be taken of the value of the perturbation which the variables are subject to at each major iteration for the gradient evaluation. In fact, from a mathematical point of view, a lower perturbation should be advisable but, at the same time, this one must induce a geometry variation in the computational domain detectable by the objective function.

5. Implementation

The optimisation code has been completely developed in the MatLab environment. Both domain definition (with Ansys ICEM-CFD 5.1) and simulation (with Fluent 6.1) have been automated resorting to batch files. In fact, all the instructions of
pre-processing, solving and post-processing have been written in log files. In this way, once the optimisation starts no other tasks are necessary until the end. Particular care has been spent in the exception management (e.g., restart of the simulation, spawning for parallel computing). Moreover, the implemented code permits us to generate a parametric model of both the domain (in terms of geometry size, number of elements, grading of the mesh, etc.) and the simulation parameters (in terms of velocity components, dissipation, number of iterations, convergence criteria, etc.); such a parameterisation is mainly related to the centreboard size and the sailing conditions. Thus, the software can be used to study different geometries in several sailing conditions with the only constraint of the topology being that only the keel without any appendages (like bulb, trim, winglets, etc.).

6. Case study

The optimisation algorithm has been applied to the design of the centreboard of the 5o5 international class racing dinghy (see http://int505.org). In the following, a brief description of the main choices is shown.

6.1. Sailing conditions

The simulations have been made under the hypothesis of close hauled sailing (reaching or running the centreboard is raised inside the trunk). The boat velocity is the one corresponding to the incipient planing that approximately happens at five knots (~2.5 m/s). The chosen centreboard cross-section is a NACA

![Drag coefficient vs. number of cells for the mesh sensitivity analysis.](image1)

![Fitness evolution and optimal planform at the 26th generation (left) and after refinement (right).](image2)
According to Section 2, the suggested real wind velocity is 3.8 m/s with a relative wind velocity of 5.8 m/s while the given leeway angle is 5°. The target lift \( L_T \), depending on the above-mentioned parameters, has been evaluated by means of Eq. (2), giving 350 N (a deflection angle \( \theta = 38^\circ \), a boom angle \( \lambda = 7^\circ \) and a sail area \( A = 15 \text{ m}^2 \) have been assumed). According to experimental data \( \text{(Fossati, 2007)} \) for a given apparent wind angle \( \alpha \) of 45°, the following values of lift and drag coefficient have been obtained \( C_L(\alpha) = 1.46, C_D(\alpha) = 0.31 \). Applying these values in Eq. (5), one can find from Eq. (4) a heeling component of about 375 N that shows an increasing value of about 8% with respect to the previous one. Finally, no heeling is considered since in a dinghy this angle is maintained close to zero by the crew; moreover, no rudder or hull lift contribution is considered due to a rapid and continuous adjustment made by the helmsman that vary the angle of attack continuously. This simplification is also conservative because in real sailing conditions the rudder contribution to the lift force is small but not negligible and thus, taking into account this simplification, a value of 350 N has been assumed for the optimisation strategy.

Note that all the numerical data (boat velocity, leeway angle, etc.) have been suggested by 5o5 World Champions and other class experts.

### 6.2. Variables and constraints

The centreboard geometry has been defined by means of a NURBS surface having \( 5 \times 50 \) control points, degrees 3 and 7, respectively, and uniform knot vectors; hence, the total number of variables is 10. The class rules \( \text{(Westell, 2006)} \) have been used to define the domain space of the design variables obtaining lower and upper bounds similar to those shown in Fig. 6.

### Table 1

<table>
<thead>
<tr>
<th>Planform</th>
<th>( L ) (N)</th>
<th>( D ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>349.7</td>
<td>31.0</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>349.6</td>
<td>29.5</td>
</tr>
<tr>
<td>Optimal</td>
<td>349.5</td>
<td>28.8</td>
</tr>
</tbody>
</table>

### 6.3. Simulation parameters

The centreboard and the related physical domain have been meshed (after several convergence tests) with a hexahedral mesh of about 235,000 cells. This size can be considered adequate; in fact, Fig. 7 shows that from about 180,000 to 350,000 cells, the difference in terms of drag coefficient is lower than 0.5%. The numerical simulations have been executed in steady state condition with a \( k-\varepsilon \) turbulent model. The wall \( y^+ \) is in the range of 10–30 and the number of iterations set at 250 (after more or less 200 iterations the residuals are lower than \( 10^{-5} \)). Each simulation takes 9 min on a PC with a dual core CPU@2.13 GHz and 2 GB RAM.

Note that most of the choices concerning the simulation parameters have been set up according to experimental test \( \text{(De Luca, 2006)} \) executed at the cavitation tunnel of the CTO (Centrum Techniki Okrętowej S.A.—Ship Design and Research Centre S.A. Gdańsk, Poland) on two centreboards showing, for a given angle of attack, a bias error between numerical and experimental results; such an error is approximately the same both for lift and drag.

### 6.4. GAs parameters

The population subject to the evolutionary process is composed over 40 individuals (whose initial estimate has been randomly chosen) and keeps a constant size. Of course, the population size is a trade between accuracy and run time; consequently, the greater is the number of processors for parallel computing, the greater could be the size. However, considering the restricted domain space and the nature of the problem we are dealing with, it can be considered a reasonable value. Since, as in nature, crossover is much more important than mutation, according to Goldberg \( \text{(1989)} \), the probability of crossover, \( p_c \), and of mutation, \( p_m \), have been chosen equal to 1 and 0.05, respectively. The quality of the individual is evaluated by means of the related fitness, whose value, to be maximized, is calculated with the inverse of Eq. (8) setting \( w_1 = w_2 = 10^3 \); such a value has been chosen in order to have a fitness of the same order of the lift coefficient.
6.5. Results

The evolution of the population has been followed for 30 generations (corresponding to 1200 different simulations). Fig. 8 shows the trend of the fitness and the optimal planform obtained at the 26th generation. This solution has been refined with a gradient-based optimisation for 10 iterations (corresponding to 270 sub-iterations) and the related shape, at the same scale, is also shown in Fig. 8. It is interesting to note how this shape recalls the one of the typical long distance soaring birds (like gulls) characterized by a high aspect ratio (O’Keefe, 2001). In Table 1, a comparison between numerical results is shown. The third column of the table gives the drag associated to three planforms (rectangular, trapezoidal and optimal) developing the target lift (shown in column two). The found optimum has a drag which is 7.1% and 2.4% lower than the one given by the rectangular and trapezoidal shapes, respectively. Fig. 9 shows, for different values of the leeway angle, the lift to drag ratio \( \frac{L}{D} \) for both the trapezoidal and optimal centreboards and the numerically evaluated efficiency \( \Delta D \) defined as

\[
\Delta D\% = \frac{D_{\text{trapezoidal}} - D_{\text{optimal}}}{D_{\text{trapezoidal}}} \times 100.
\]

The optimal wing has always a greater ratio, \( \frac{L}{D} \), than the trapezoidal while the difference is the greatest for the imposed leeway angle of 5°.

Fig. 10 shows the contours of the velocity magnitude (left) and those of the turbulence intensity in percentage (right) behind the trapezoidal and optimal centreboards. From this figure, it can be observed that the optimal centreboard has a smaller zone affected by turbulence close to the upper side of the trailing edge.

Finally, Fig. 11 shows the manufactured centreboard made with a wood core (CNC machined) and two plies of woven carbon and epoxy resin which will be adopted by ITA 4970 during the next 505 world championship.

7. Conclusions

In this paper, a numerical approach to the keel design of a sailing yacht has been described. The method uses of commercial and \textit{ad hoc} developed software. An optimisation procedure mainly based upon GAs permits us to find the optimal planform of a keel according to the results of the CFD simulations. The software, which is fully automatic, has been parameterised and hence can be used to study different geometries in several sailing conditions. The case study has shown the reliability and the robustness of the implemented code.

![Fig. 10. Comparison of velocity magnitude (left) and turbulence intensity close to the root (right) of the optimal and trapezoidal centreboard.](image)

![Fig. 11. The manufactured centreboard made with a CNC wood core and two plies of woven carbon and epoxy resin.](image)
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